Revealed Preference Theory

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1 Chp 3 : Rational Demand - Basic Setup

1.1 Weak Rationalization

Let X be a convex consumption space and consider demand function $d(p, m)$ where:

- $p = (p_1, p_2, \ldots, p_n)$ is the price vector
- m is income
- $d(p, m) = \{x \in X : p \cdot x \leq m \text{ and } (y \geq x) \Rightarrow (p \cdot y > m)\}$

1.2 Consumption Dataset

A consumption dataset (x^k, p^k) for $k = 1, 2, \ldots, K$ consists of:

- x^k is the "consumption bundle" at prices p^k purchased
- Total income devoted to consumption

For each k, x^k is "consumption bundle" at prices p^k.

2 Key Axioms and Theorems

2.1 Preference Relations

Let $\succsim$ denote the preference relation that carries demand of x: $x \succsim y$.
Key Insight: "x is at least as good as y"

2.2 GARP - Generalized Axiom of Revealed Preference

There exists some preference relation $\succsim$ such that:

- If $B \in B$, $x \in C(B) \Rightarrow x$ is $\succsim$-maximal in B

Every chosen item is at least as good as all other available items in that choice set.

Note: This type of rationalization has too little - the observed choices does not have to make sense to other situations.

2.3 Weak Axiom of Revealed Preferences (WARP)

If a consumer picks bundle x over y when both are affordable, she shouldn't pick y over x in another situation where both are affordable.

Mathematically:

1. Monotonicity: Rule out indifference in some cases
2. If consumer spent more on x_1 even though x_2 was available for less, she cannot be indifferent between x_1 and x_2. She must strictly prefer x_1.
3. No cycles - must exactly pick chosen bundle

Consumer would've picked x_2 and used leftover money to buy more of something (x_*).

If x_1 was chosen over x_*, the consumer strictly prefers x_1.

3 Afriat's Theorem

Let X be a convex consumption space and $D = \{(x^k, p^k)\}_{k=1}^K$ be consumption dataset. Then:

1. D has a locally non-satiated weak rationalization
2. D satisfies GARP
3. There are strictly positive numbers U^k and λ^k for each k s.t.

$$U^k \leq U^\ell + \lambda^\ell p^\ell \cdot (x^k - x^\ell) \text{ for each pair } (k, \ell)$$

4. D has a continuous, concave & strictly monotonic rationalization $u : X \to \mathbb{R}$

3.1 Key Points

1. There exists at least one preference relation that is rational, locally non-satiated/continuous such that for each observed choice x_k is at least as good as any affordable alternative at p_k.
2. Data satisfy GARP (no loops)
3. Rationalizability
4. You can construct a nice utility function

4 Applications

4.1 Afriat's Theorem Applications

If you look at a person's shopping data and see no loops (GARP), then not only is the person behaving rationally, but you can invent a "score card" - a utility function that explains what they did.

Critics argue: The edge this might marginalize to build such a rule.

4.2 Budget Sets

A set of all bundles a consumer can afford at a given price & income:

$$B = \{x \in X : p_k \cdot x \leq I\}$$

4.3 Generalized Afriat's theorem - 4 way equivalence

Finite dataset $D = \{(x_k, B_k)\}_{k=1}^{K}$

Each B_k is defined as above.

1. Locally non-satiated weak rationalization
2. GARP holds
3. Generalized inequalities are feasible
4. There is cardinally utility U - continuous monotonic function

If $X \to \mathbb{R}$, then x_k is maximizer of $u(x)$.

5 Abstract Cases

Suppose people face more complicated constraints than energy & prices.

$g_k : X \to \mathbb{R}$ is continuous and monotonic.

If $x \leq x'$ (component-wise), then $g_k(x) \leq g_k(x')$ and the chosen bundle x_k such that $g_k(x_k) = 0$.

$$B_k = \{x \in X : g_k(x) \leq 0\}$$

6 Revealed Preference Relations

6.1 Direct Revealed Preference ($\succ_R$)

"Bundle x is revealed preferred to y" if in observation k:

- Consumer chooses $x = x_k$ and y was affordable
- $\therefore x \succ_R y$

6.2 Applying GARP - Sequence of Revealed Preference

Choices cannot form a cycle in at least one strict step.

7 Key Takeaways

1. Revealed preference theory allows us to test whether observed behavior is consistent with rational choice theory
2. GARP is the key testable condition - no preference cycles
3. Afriat's theorem provides the bridge between observable behavior and underlying preferences
4. The approach is non-parametric - we don't assume specific utility functional forms
5. Real-world applications include testing consumer rationality using household expenditure data

8 Strong Rationalization

8.1 Definition

The same set of preferences can completely explain all the choices made over all possible sets of options.

Formally: A preference relation strongly rationalizes data $D = \{(x^k, p^k)\}_{k=1}^K$ if for every k and every bundle y (where $y \neq x^k$) that was affordable at p^k:

$$p^k \cdot y \leq p^k \cdot x^k \text{ and } y \neq x^k \Rightarrow x^k \succ y$$

8.2 Strongly Revealed Preference ($\succ_S$, $\succsim_S$)

$x \succ_S y$: There is an observation where x was picked and y was affordable.

$x \succsim_S y$: Either $x \succ_S y$ or $x = y$.

8.3 SARP (Strong Axiom of Revealed Preference)

There are **no** cycles in the strongly revealed preference ($\succ_S$, $\succsim_S$) such that at least one step is strict.

If you can go from $x_1 \to x_2 \to x_3 \to \ldots \to x_n = x_1$, this is a contradiction.

9 Strong Afriat's Theorem

9.1 Statement

Locally non-satiated & strongly rationalizing preference relation $\succsim$ satisfies SARP $\Leftrightarrow$ exist positive numbers U^k, λ^k s.t. for all k, ℓ:

$$U^k \leq U^\ell + \lambda^\ell p^\ell \cdot (x^k - x^\ell) \quad \text{and} \quad U^k \leq \lambda^k$$

Strictly concave & monotonic utility function U that uniquely rationalizes the data.

10 Smooth Utility

A utility function assigns a real number to each possible number of goods.

Differentiable utility $\Rightarrow$ a utility function is differentiable if it has a well-defined partial derivative with respect to each good at every point in domain (no sudden jumps/sharp corners).

Marginal utility (the benefit from consuming a bit more) changes smoothly $\Rightarrow$ if $\exists$ a continuous $\frac{\partial U}{\partial x_i}$.

Key Insight: Sometimes the data can force a kink - non-differentiable point in any utility function.

11 The Theorem (Chiappori & Rochet)

Dataset $D = \{(x^k, p^k)\}_{k=1}^{K}$ can be rationalized by a smooth, strictly monotonic & strictly concave utility $\Leftrightarrow$

1. The data satisfies SARP
2. No 2 different bundles are chosen at the same price vectors

If $x^k \neq x^\ell$ at $p^k = p^\ell$, then the utility must have a kink. Otherwise, you can fit a smooth utility.

12 General Budget Sets

Not all consumption constraints come from linear budgets. Budgets can arise from nonlinear prices, discounts.

B_k are more general sets.

12.1 General Structure

(Set of impossible bundles cannot be: can't buy more than a household)

1. Complement of B^c in X is a convex set
2. If $x \in B$, $y \gtrsim x$, then $y \notin B^c$
3. If $y \succ x^c$, then $y \notin B^c$

If bundle x is outside the budget set and $y \gtrsim z$, then y won't belong to B^c.

$\Rightarrow$ If $y \succ x^k$ and not in B_k, spend always consume. Go much as possible, spend always.

13 Theorem

A dataset where observations are pairs (x^k, B^k), if & only if it satisfies SARP.

13.1 Partially Observed Prices & Consumption

Now you see all prices but quantity $m + n$, you only observe m.

No model what the observed (x^k, p^k) data is, you can always come up with hypothetical assumptions & higher goods that make data rationalizable.

Partial price: Reasonable if each hidden bundle stops is rationalizable.

Partial consumption: Always rationalizable.

14 Revealed Preference GARP

Given a dataset of observed data & price vectors:

1. Each observation (x^k, p^k) - x_k chosen bundle, p_k price vector

2. K observations

A revealed preference graph should for each decision - which other decisions the consumer preferred they used less, based on what was affordable & what they picked.

15 Extensions and Applications

15.1 Pattern of Revealed Preference (”Arrow”)

Any pattern of revealed preference can be considered in the right bundles/prices, if you choose the consumption space freely.

For $\mathbb{R}^n_+ \supseteq 2$ extremes - total strict preference & only reflexivity are always feasible.

Significance: Revealed preference encode all observable policy logic in a dataset. In high dimension, any such graph can occur.

16 Limitations and Extensions

1. **Strong assumptions:** Perfect information, no mistakes, stable preferences

2. **Limited predictive power:** Can test consistency but limited forecasting ability

3. **Extensions:** Stochastic revealed preference, collective household models, behavioral extensions

4. **Practical challenges:** Economists can't observe every price or every aspect of consumption

5. **Real-world complications:** Dataset weakly rationalizable if for every group of observations showing the same hidden consumption, the observed full demand price are themselves weakly rationalizable

16.1 Chp 7: Stochastic Preferences - Basic Setup

Suppose we have a finite set X of options. When a person faces a menu (subset) $A \subseteq X$ of options, they might not always pick the same thing.

For any menu $A \subseteq X$ and $x \in A$:

$$P_A(x) : \text{Probability of picking } x \text{ from } A \tag{1}$$

Since probabilities must add to 1: $\sum_{x \in A} P_A(x) = 1$

16.2 Interpretations of Stochastic Choice

Population level: $P_A(x)$ is the fraction of people who would pick x if offered A.

Individual level: For a single person who is not consistent, $P_A(x)$ is the probability of them choosing x when A is offered.

16.3 Where do these probabilities come from?

Assume each agent has a strict preference order over X. All possible preference rankings over X are Π.

There is a probability distribution ν over all these possible strict preference rankings.

When choosing from A, the agent picks their favorite from A according to their preference ranking as per their "play."

17 Characterizing Rationalizable Systems

To know if a system $\{P_A(x)\}$ observed choice probabilities can be explained by such a rational model:

17.1 Axiom of Revealed Stochastic Preference (ARSP)

▷ A cost condition relating preferences after diet minus...

18 Block Massinal Polynomials

For each $x \notin A$ and each subset $A \subseteq X$:

$$x \in A^c, x \notin A \tag{2}$$

$$K_{x,A} = \frac{|A|}{C_0}(z-1)^{|A|-1} \leq P_c(x) \tag{3}$$

▷ These measure the net change that x is picked from A and its submenus. If any $K_{x,A} \leq 0$, then the system isn't rationalizable.

18.1 Interpretation

$K_{x,A}$ = probability (according to random preference distribution) that every element in A is ranked higher than x and x is the best among everything else.

19 Luce's Model

Luce's model → probability of drawing any specific alternative is proportional to its utility relative to total utility of all options.

Luce's model describes how people make decisions when they have several choices.

Basics: - Probability of choosing an alternative = "value" or "utility" item from alternatives.

19.1 Independence of Irrelevant Alternatives (IIA)

▷ The relative odds of choosing between 2 alternatives are unaffected by other available alternatives.

Set of alternatives: $X = \{x_1, x_2, \ldots, x_n\}$

Pick $x \in A$: By IIA, the choice probability for x is unaffected by the size of choice set A.

$$\frac{P_A(x)}{P_A(y)} = \frac{P_B(x)}{P_B(y)} \tag{4}$$

for any subsets A, B of A (assuming $x, y \in A \cap B$).

19.2 Mechanics

Utility ↔ Probability:

Each alternative has utility $U(x)$:

$$P_A(x) = \frac{U(x)}{\sum_{y \in A} U(y)} \tag{5}$$

20 Random Utility Models

20.1 Relation to RUM

RUM = framework for modeling decisions which assigns a utility to each option.

Core idea: each alternative has utility $U(x)$:

$$U(x) = V(x) + E(x) \tag{6}$$

Systematic part → deterministic component considers across simulations

Random part

→ introduces

variability

Random expected utility

Choices are not direct alternatives but "lotteries" over a finite set of outcomes Y. Y occurs with some probability.

x is a collection of all possible lotteries over Y. Each lottery is a probability distribution over these outcomes.

A utility function u is a vector in $\mathbb{R}^Y$, where each element corresponds to utility assigned to each prize.

An agent chooses an maximizing their expected utility x over Y:

$$U \cdot x \geq U \cdot y \tag{7}$$

▷ Agents choose the option that offers highest expected utility.

21 System of Choice Probabilities

Is expected utility rationalizable iff it is monotone, linear, extreme & mixture continuous.

→ Here X is infinite countable or probability over outcomes, but decisions are considered over finite/non-empty subsets.

In (A, x) set of utility vectors that make "option x at least as good as any other option in A":

$$P(A) = 4^{-N(A,x)} \tag{8}$$

▷ Probability of choosing x from A is determined by probability measure μ over all rationalizing utilities.